PREDICTION OF UNSTEADY AERODYNAMIC FORCES VIA NONLINEAR KERNEL IDENTIFICATION

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Abstract

Aeroelastic studies play a critical role in aircraft safety and design and, to accelerate the design process and reduce life cycle costs, nonlinear aerodynamic effects must be considered from the onset. The Volterra theory of nonlinear systems provides a mathematically rigorous approximation technique to describe these unsteady aerodynamic effects. A critical problem, however, is the difficulty of identifying the Volterra kernels. The present paper demonstrates the use of a time-domain Volterra kernel identification method which uses physically realizable inputs, is robust with respect to noise, and minimizes or eliminates the need for analytical assumptions. This technology provides a rational means of simulating nonlinear aerodynamic behavior in multidisciplinary analyses and will facilitate the incorporation of high-fidelity tools into the preliminary design phase of aerospace vehicles.

1 Introduction

Recent results [1] have demonstrated the feasibility of developing a new Volterra kernel identification tool suitable for aeroelastic analysis from wind-tunnel or flight-test data. The method is based on the well-established Volterra-Wiener theory [2,3] of nonlinear systems and uses regularization techniques [4] to create a versatile and practical tool for uncovering and formulating nonlinear reduced-order models of aeroservoelastic systems. These models can be derived from experimental observations and numerical models alike, and they have the potential to combine speed and high accuracy, making them suitable for control system embedding and nonlinear control system design.

A low-order version of the time-domain Volterra kernel identification method has been assembled into a prototype nonlinear identification and prediction software system. This system uses physically realizable inputs, is robust with respect to noise, and minimizes or eliminates the need for analytical assumptions. The problem of indirect kernel identification (i.e., the inference of the kernels from experimental observations) is, fundamentally, an improperly posed problem. This improperly posed problem arises as a result of having to solve integral equations analogous to those appearing in inverse scattering problems, and inverse problems in general. The solution method used herein expands on a unique extraction technique previously developed [5] for the identification of nonlinear indicial kernels. This technique has been applied to wind-tunnel data from a maneuvering delta wing configuration [6]. Its application to the identification of Volterra kernels in the case of time-invariant aeroelastic systems is shown here.

Although there are a number of Volterra kernel identification techniques in the literature, the present method has the unique ability to extract the kernels using physically realistic excitations. This feature makes it ideally suited for use with existing wind-tunnel or flight-test data.

2 Objectives

The objective of this work was to demonstrate the feasibility of developing a Volterra kernel identification tool for aeroelastic analysis using wind-tunnel and/or flight-test data. The ultimate objective is to provide engineers and scientists with a new nonlinear modeling capability to analyze the aeroservoelastic characteristics of flight vehicles and to formulate economical and physics-based reduced-order models suitable for control system design. The specific objectives of the study were to demonstrate the second-order kernel version of the method, and to show that the method is accurate, robust, and can be extended to higher-order kernels. The present paper focuses on the application of the method to unsteady aerodynamic data from a pitching wing.

3 Background

Aeroelastic studies, such as the prediction of flutter boundaries and limit cycle oscillations, play a critical role in aircraft safety and design. To perform these analyses with the required accuracy, nonlinear aerodynamic effects must be included. The Volterra theory of nonlinear systems addresses the need for efficient and accurate reduced-order modeling of these unsteady aerodynamic effects. A critical problem, however, is the difficulty of identifying the Volterra kernels. The present paper demonstrates application of a new identification method that is suitable for use with wind-tunnel and/or flight-test data.

3.1 Importance of Volterra Kernel Identification to Aeroelasticity Studies

The need to incorporate nonlinear aerodynamic effects in aeroelasticity studies of flight vehicles is well recognized [7-10]. Nonlinear aeroelastic effects can arise from either the structure or the flow. Structural nonlinearities may be either of a geometric nature or of material origin. Aerodynamic nonlinearity arises from a variety of sources, such as shock motion, the appearance or disappearance of shock waves, and locally separated and vortical flow. The work of Reference 7, for example, shows that, even for the prediction of flutter onset, aerodynamic nonlinearity may have to be taken into account for a wide range of conditions.

In order to accurately characterize nonlinear unsteady aerodynamic effects in aeroelasticity, the application of the Volterra theory of nonlinear systems was recently proposed [11-14]. The Volterra series approach provides a mathematically rigorous approximation technique to describe nonlinear systems. This theory asserts that a time-invariant nonlinear system can be modeled as an infinite sum of multidimensional convolution integrals. For weakly nonlinear systems, only a few terms in the series need to be modeled. In the particular case of a truncated second-order system, an additional advantage is that the system may be represented using a bilinear state space formulation which is amenable for use with modern control systems [14] and, therefore, aeroservoelastic analyses. This approach has previously been shown to be well-suited [11] to the incorporation of nonlinear CFD results into aeroservoelastic analyses. It will be argued here that the Volterra theory is also well-suited to the accurate characterization and efficient modeling of experimental data (i.e., from wind-tunnel models or flight vehicles), for the purpose of performing vibration, aeroelastic, and aeroservoelastic studies. These benefits, along with the solid mathematical foundation upon which the Volterra series approach is based, are of considerable interest for the modeling of aeroelastic phenomena.

3.2 Difficulty of Volterra Kernel Identification

While the Volterra theory of nonlinear systems is well-established (Volterra [2], Wiener [3]), as a tool it has received little attention outside electrical engineering and apparently none in aeroelasticity studies, until the last decade or so. The main reason for this has to do with the ease of use of linear methods. However, with the improvement of computational methods and noticeable increases in computing power, higher-fidelity (i.e., nonlinear) models are presently being sought.

Another reason for the previously limited use of Volterra theory is the inherent difficulty of identifying the Volterra kernels. As previously mentioned, the Volterra series is made of a sequence of multidimensional convolution integrals, and each is associated with an input independent “kernel” function. Once the kernels are known, it is formally a straightforward matter to compute the truncated series for any arbitrary schedule of the system's input. Thus, the problem of kernel identification is central to the usefulness of the method.

Unfortunately, in many cases kernel identification happens to be extremely difficult. One exact technique, applicable to discrete-time systems, is the use of unit-sample responses. Such an approach was successfully used by Silva [11] to identify kernels from CFD codes. However, this “direct” identification method does not generalize to kernel identification from experimental data. In fact, there appears to be an inverse relationship between accuracy and physical realizability amongst existing Volterra kernel identification schemes. Dual-pulse digital filter techniques [11], for example, have been shown to very accurately identify the second-order Volterra kernel associated with the computed plunging motion response of an airfoil in transonic flow. However, inertial effects preclude attempting this kind of excitation in wind-tunnel tests (much less, flight tests). Ingenious methods for “measuring” Volterra kernels from experimental data have been devised over the years, for example: the white noise/cross-correlation method of Lee and Schetzen [15], or Boyd et al.'s [16] multitone harmonic probing technique for kernel separation. In all cases, however, these experimental approaches require that the nonlinear system under consideration be subjected to very specific excitations, many of which are not applicable to wind-tunnel models and, indeed, impossible for flight tests.

Thus, an alternative method is required that would allow the identification of the Volterra kernels from unsteady aerodynamic data. The requirements for such a method are as follows: the method must use physically realizable inputs, must be accurate and robust, and must be applicable to existing data (i.e., data not specifically generated for the purpose of kernel identification), including flight tests. In addition, the method should minimize or eliminate the need for analytical assumptions about the underlying kernel(s).

3.3 Method

The proposed method is based on a newly developed extraction technique which has been successfully applied to the identification of nonlinear indicial kernels [6] from an experimental database. The details of the theory are described in Reference 1. A brief summary is given below.

Let x(t) designate the input to the system (for example, a control surface displacement or modal amplitude). Let y(t) designate the output of the system (for example, a generalized aerodynamic force). The Volterra series expansion of y with respect to x is given by

$$y(t) = \sum_{n=1}^{\infty} \int_0^t \cdots \int_0^t K_n(t-\tau_1, t-\tau_2, \cdots, t-\tau_n)\, x(\tau_1) \cdots x(\tau_n)\, d\tau_1 \cdots d\tau_n$$

where K_n is the nth-order Volterra kernel. The process of Volterra kernel identification falls in the general category of inverse problems, since the objective is to determine the *internal structure* of the physical (aerodynamic/aeroservoelastic) system under consideration, based on the system's measured behavior. The method takes advantage of the fact that, although the truncated Volterra series is nonlinear with respect to the input parameter x, the inverse problem (the problem of identifying the Volterra kernels) remains linear with respect to the kernels. The approach [1] consists of expanding the unknown kernels on some known basis function set,

$$K_n(t_1, t_2, \ldots, t_n) = \sum_{k=1}^{N_{nD}} c_k^{(n)} \mu_k^{(n)}(t_1, t_2, \ldots, t_n)$$

and solving for the unknown basis function coefficients $c_k^{(n)}$. Multiple measurements $\{x(t_i),y(t_i)\}$ of the system's behavior are then interpreted as constraints which the unknown basis function coefficients must simultaneously satisfy. This results in a linear system of equations which is typically ill-conditioned. The task of solving this rank-deficient discrete problem is then handled using one of several available regularization methods [4,17].

For a more detailed account of the method, the interested reader is referred to Reference 1. For the purpose of the application described here, it suffices to mention that this method was designed to be robust with respect to noise, and to provide good accuracy without sacrificing physical realizability. Therefore, the method should be applicable to wind-tunnel and flight-test data. In particular, it will extend the usefulness of these data by allowing the formulation of efficient and compact models of unsteady nonlinear aerodynamic effects for aeroelastic and aeroservoelastic analyses.

4 Results

The extraction method of Reference 1 is used here to identify the first- and second-order Volterra kernels associated with the aerodynamics of a wing which is dynamically pitched about its quarter chord location. The wind-tunnel model is considered to be rigid. The wing had a rectangular planform, with a constant thickness NACA0015 airfoil profile. The model was instrumented with a total of 45 pressure taps, divided equally among three span locations: 0%, 37%, and 80% of the total wing span. The experiment was carried out in a low-speed wind-tunnel at the U.S. Air Force Frank J. Seiler Laboratory [18]. A variety of forced pitching motions were carried out. In addition to the time dependent pressure profiles, time history data for the following sectional force coefficients are available: lift coefficient *cl*, drag coefficient *cd*, normal force coefficient *cn*, tangential force coefficient *ct*, and pitching moment coefficient *cm*.

These forced unsteady aerodynamic data have been the topic of several papers by Faller and Schreck and coworkers [18-20] emphasizing the physics of three-dimensional dynamic stall. However, the interest here is in applying the Volterra identification tool developed in this study to a nonlinear "plant" where the nonlinearities are of an aerodynamic nature. The pitching wing database constitutes such a plant. In addition, the use of the data is made particularly convenient, due to the existence of several neural network programs which have been trained to reproduce many aspects of the data with good fidelity. The neural network used in this study was shown [20] to replicate not only the constant pitch rate data it was trained on, but also to be able to predict the flow responses to novel maneuvers (both constant and variable pitch rates) with surprisingly high accuracy. For our purpose, the trained neural network can be considered as a "black box" representation of the time-dependent loads and pressure distributions. This black box representation is accurate within a reasonably wide parameter space, and it will be assumed that the flow responses predicted by the model are "as good" as if they were directly measured in wind-tunnel tests.

For all of the results presented below, the wing was first pitched up from 0° angle of attack to a post-stall angle of 17° at a nondimensional pitch rate based on wing chord and freestream velocity of $\alpha^+ = 0.04$. The angle of attack was then held constant for approximately 35 convective times, so as to eliminate all prior transients associated with the dynamic stall event. This new reference condition ($\alpha = 17°$) was chosen because of the known highly nonlinear behavior around this condition [21]. Starting from this new reference point, the wing can perform various small amplitude motions, and it is the result of these motions which is analyzed to extract the Volterra kernels. An example of such a motion is shown in **Figure 1**. Unless otherwise specified, the results presented hereafter all pertain to the sectional lift coefficient *cl* at 0% wing span.

The basis functions were chosen to be decaying exponentials. Nine exponential basis functions (inverse time constants ranging from 0.5 to 4.5) were used for the first-order kernel. For the two-dimensional kernel, either 5×5, 7×7, or 10×10 matrices of time constants in a similar range were used [1].

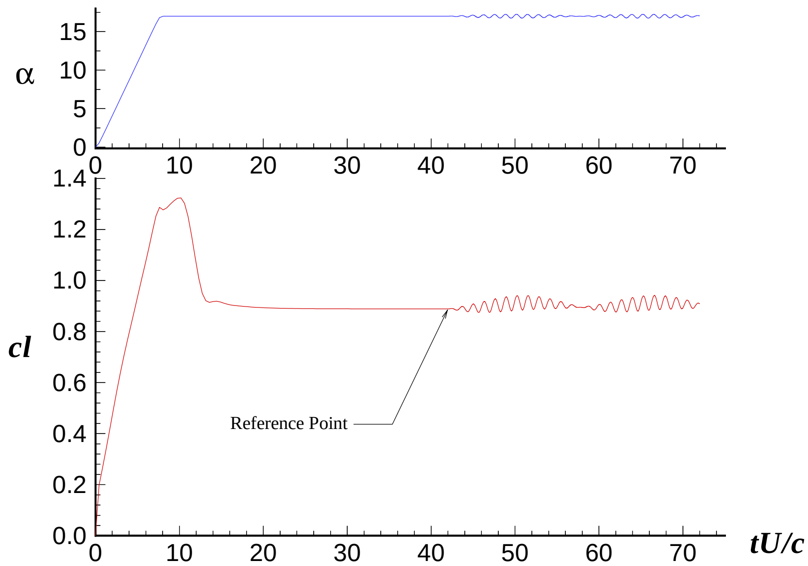

Fig. 1 Definition of Reference Condition for Pitching Wing Data Analysis.

As previously mentioned, the difficulty of system identification stems from the fundamentally ill-posed nature of linear inverse problems. This point cannot be overemphasized. The extraction method that was developed has the desirable characteristics of robustness and ease of use, thanks to the use of regularization techniques [4,17]. However, the possibility always exists that the results of the extraction could amount to little more than a sophisticated form of data fitting. Therefore, in order to ensure that this is not the case and that the true underlying kernels have been identified, several quality assurance (verification) steps must be taken.

- The first of these steps is to check that the data used for identification (also referred to as the training data) can be repredicted using the extracted kernels. Although this is only a necessary, not a sufficient, condition, this requirement is not trivial.

- The second verification step is to check that the linear impulse response resulting from the simultaneous identification of the first- and second-order kernels using nonlinear data matches the result of a first-order-only extraction based on linear data. In practice, it may be very difficult to know with certainty the extent to which the data may be considered linear. Therefore, this second step may not always be applicable. In the present test, however, such data *are* available and were used to further the confidence in the results

- The third test is the convergence test. It must be shown that the results converge to the same kernels as the training data set is progressively enriched. Similarly, one should verify that the results are minimally sensitive to the details of the basis functions chosen.

- The fourth and most important test of the method is to verify that novel data (data not included in the training) can be predicted on the basis of the extracted kernels.

Two sets of results will be presented. The first set corresponds to the case where the kernel extraction was carried out using only two wing motions. This case was used to conduct a parametric study illustrating convergence with respect to the basis function set. It was later realized, however, that these extracted kernels did not predict with sufficient accuracy several novel data sets, including simple harmonic data at specific frequencies. In other words, these kernels failed test #4 (insufficient training). This problem was remedied in the second set of results, in which the kernels were extracted on the basis of 17 training data sets. Although full convergence has not been attained, this second set of results are used to document the cross-predictive ability of the extracted kernels.

4.1 Basis Functions Convergence

Two integrated multistep [22] (wide bandwidth) inputs were used to simultaneously identify the first- and second-order kernels of the system. The extraction was repeated three times, using respectively 5×5, 7×7, and 10×10 two-dimensional basis functions for the second-order kernel. The extracted first-order kernel was shown to be insensitive to the choice of two-dimensional basis functions (**Figure 2**). The extracted second-order kernels are shown in **Figures 3–5**.

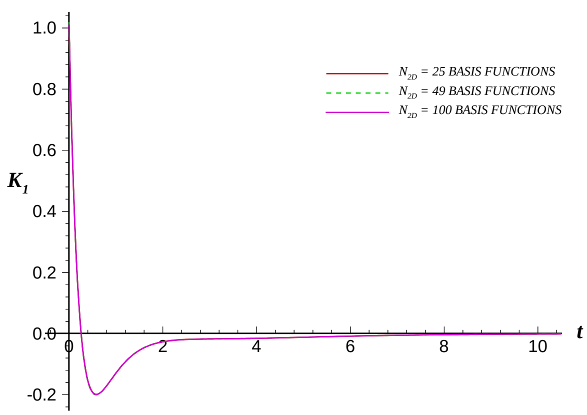

Fig. 2 Convergence of Extracted First-Order Volterra Kernel as a Function of Two-Dimensional Basis Set.

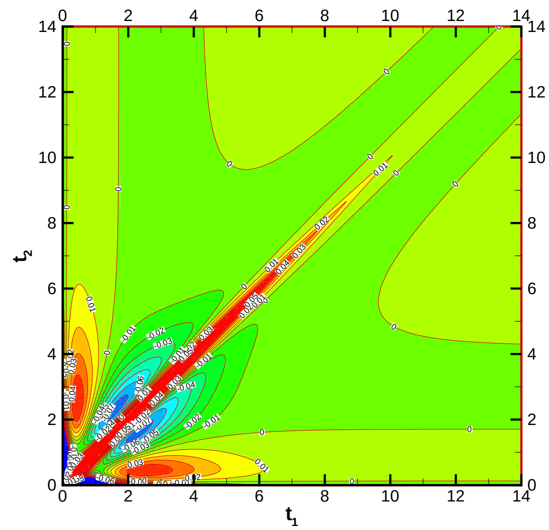

Fig. 3 Extracted Second-Order Kernel Using 25 Basis Functions.

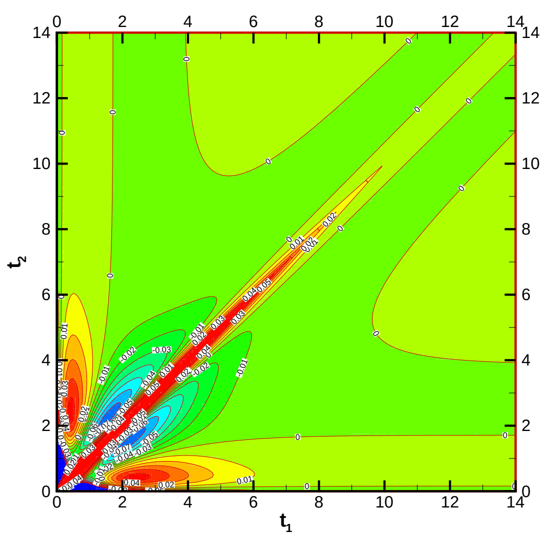

Fig. 4 Extracted Second-Order Kernel Using 49 Basis Functions.

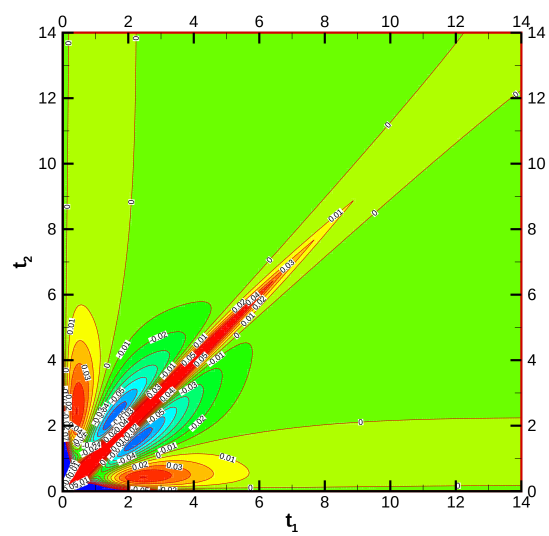

Fig. 5 Extracted Second-Order Kernel Using 100 Basis Functions.

Figures 2-5 illustrate the robustness of the results, since both first- and second-order Volterra kernels are minimally affected by the choice of basis functions, despite a large increase in the number of effective degrees of freedom. To facilitate the comparison between kernels (**Figure 6**), the data shown in **Figures 3-5** are reduced using four one-dimensional representations of the kernel (radial cuts of the symmetric two-dimensional kernel at fixed geometric angles) in lieu of one two-dimensional representation.

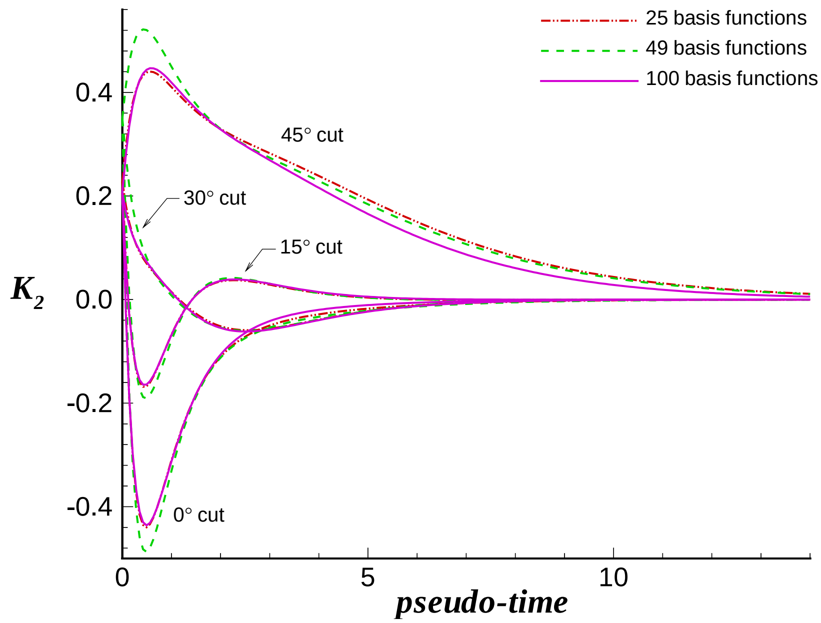

Fig. 6 Comparison of Extracted Second-Order Kernels Along Four Radial Cuts.

The comparison of radial cuts is a more sensitive representation of the differences between the various two-dimensional kernels. Although the kernels are not identical, **Figure 6** displays good convergence properties. In addition, it was shown [1] that the basis function coefficients themselves have a certain element of structure and appear to converge to a well-defined continuous spectrum.

4.2 Prediction Capability

In this second set of results, the first- and second-order Volterra kernels were extracted using a training data set which included:

- two small amplitude (0.03˚) integrated multistep inputs;
- a small matrix of sinusoidal excitations $\alpha(t) = A \sin(\omega t)$; this matrix was obtained from the combinations of three amplitudes (A = 0.01˚, 0.03˚, and 0.1˚) and five angular frequencies (ω = 0.01, 0.02, 0.1, 0.5, and 3.0).

Thus, all of the training motions have an amplitude less than 0.1˚ (0.2˚ peak-to-peak).

Although the extracted kernels are not believed to be fully converged, they can be used to illustrate the predictive ability of the model developed thus far. The following figures show that the extracted kernels are capable of reproducing accurately not only the training data, but also novel data of a different character.

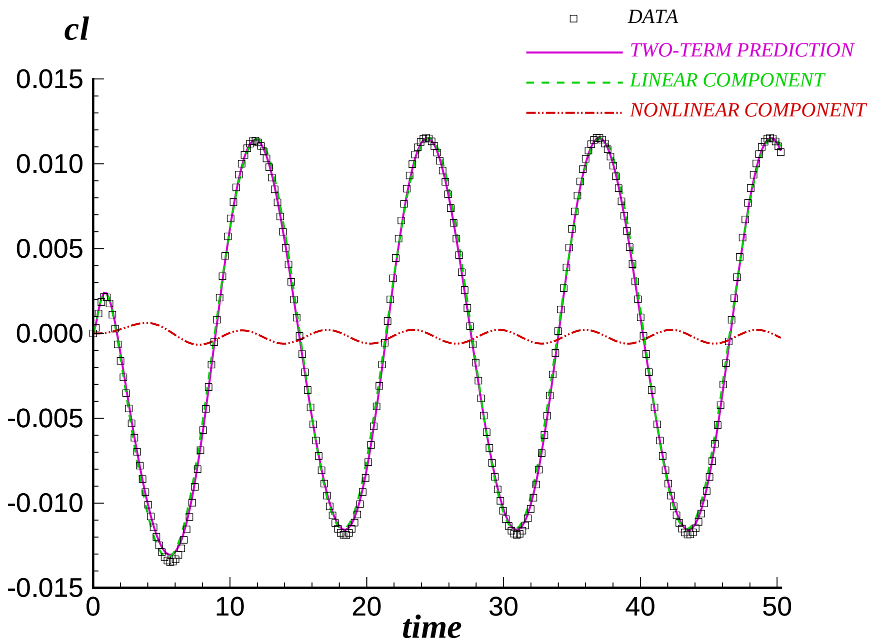

Fig. 7 Reprediction of Training Data (A = 0.1°, ω = 0.5) From Extracted Kernels.

Figure 7 shows the truncated two-term Volterra series prediction for one of the sinusoidal training data sets (A = 0.1˚, ω = 0.5). In this figure, the data are represented with symbols; the various lines represent respectively the linear component of the prediction (i.e., the result of the first-order convolution), the nonlinear component of the prediction (i.e., the result of the second-order convolution), and the total prediction, which is the sum of the two. The total prediction is in good agreement with the data. The ability to predict novel data is illustrated in **Figure 8**.

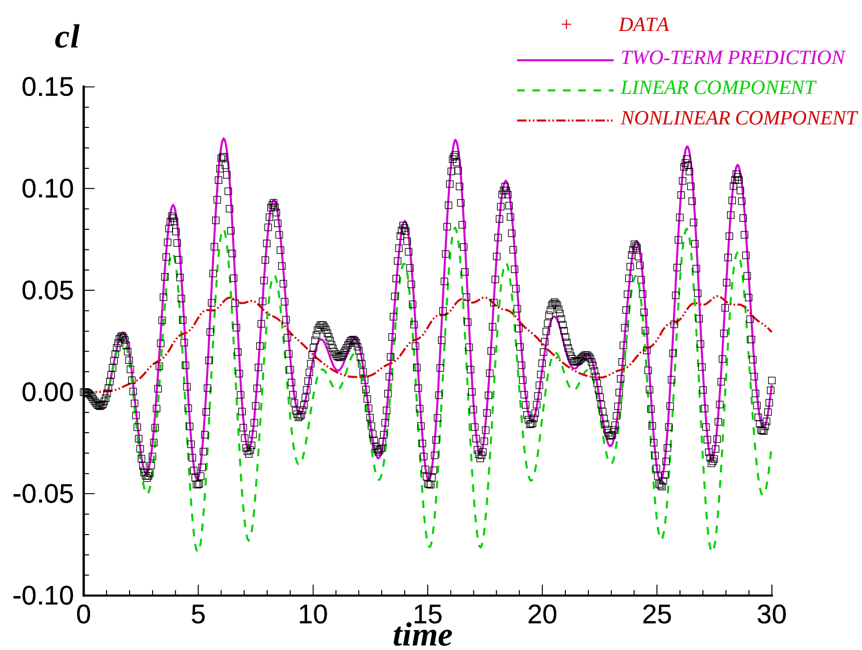

Fig. 8 Novel Data Prediction From Extracted Kernels ; Input: α = 0.5° sin(0.3t) sin(2.8t + π).

The total prediction (labeled *TWO-TERM PREDICTION*) is also seen to be in good agreement with the data. The amplitude of the oscillations is five times the largest amplitude used in training. Equally good agreement was shown at lesser amplitudes, while the accuracy of the prediction was found to deteriorate slightly at 0.7° amplitude. This is consistent with the a posteriori examination of the static nonlinearity of the system (not shown), which revealed that the point at which a quadratic fit ceases to be an accurate approximation of the nonlinear response is around $\alpha_{static} = 0.5°$. Thus, the data of **Figure 8** correspond to the farthest conditions from the training data at which one could expect a two-term truncation of the Volterra series to adequately describe the system. Reference 1 reports similar results for sectional pitching moment coefficient *cm*. The agreement demonstrated in **Figure 8** shows that first- and second-order Volterra kernels extracted from primarily oscillatory data can be used to predict novel data whose parameters lie far outside the training range.

5 Conclusions

A feasibility study of an aerodynamic data-based Volterra identification system has been conducted. A preliminary version of the method, capable of extracting first- and second-order Volterra kernels from unsteady data was successfully demonstrated. The core technology of this system is based on the well-established Volterra-Wiener theory of nonlinear systems, coupled with parameter identification techniques. The present paper presented an application of the method to the nonlinear unsteady aerodynamics of a wing undergoing oscillations in pitch. The developed method can be used to extract Volterra kernels using physically realizable inputs and is, therefore, suitable for use in aeroelastic studies from experimental data. The extraction technique has been shown [1] to be accurate, convergent, and robust, and to admit formal extensions for the identification of higher-order kernels. The technology has the potential to provide scientists and engineers with an economical and practical tool to characterize the nonlinear aeroservoelastic response of flight vehicles during the conceptual, preliminary, and final design phases.

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7 References

1. Reisenthel, P. H., "A Nonlinear Volterra Kernel Identification System for Aeroelastic Applications," Nielsen Engineering & Research, NEAR TR 547, Jun. 1999.
2. Volterra, V., *Theory of Functionals and of Integral and Integro-Differential Equations*, Blackie & Sons Ltd., London, 1930.
3. Wiener, N., "Response of a Non-linear Device to Noise," Report V-165, Radiation Laboratory, Massachusetts Institute of Technology, Cambridge, MA. Available as U.S. Department of Commerce publication PB-58087, Apr. 1942.
4. Hansen, P. C., *Rank-Deficient and Discrete Ill-Posed Problems*, SIAM, Philadelphia, 1998.
5. Reisenthel, P. H. and Bettencourt, M. T., "Extraction of Nonlinear Indicial and Critical State Responses from Experimental Data," AIAA 99-0764, Jan. 1999.
6. Reisenthel, P. H. and Bettencourt, M. T., "Data-Based Aerodynamic Modeling Using Nonlinear Indicial Theory," AIAA 99-0763, Jan. 1999.
7. Murty, H. S., "Nonlinear Aspects of Transonic Aeroelasticity," University of Toronto, UTIAS-RN-345, Jan. 1993.
8. Chen, W. C. and Dugundji, J., "A Formulation of Nonlinear Limit Cycle Oscillation Problems in Aircraft Flutter," AFOSR-TR-97-0030, Sep. 1993.

9. Becker, J., "Aeroservoelastic Stability of Aircraft at High Incidence," *Maneuvering Aerodynamics*, AGARD-CP-497, 1991.
10. Cunningham, A., "The Role of Non-Linear Aerodynamics in Fluid-Structure Interactions," AIAA 98-2423, Jun. 1998.
11. Silva, W. A., "Identification of Linear and Nonlinear Aerodynamic Impulse Responses Using Digital Filter Techniques," AIAA 97-3712, Aug. 1997.
12. Silva, W. A., "Application of Nonlinear Systems Theory to Transonic Unsteady Aerodynamic Responses," *Journal of Aircraft*, Vol. 30, No. 5, 1993, pp. 660-668.
13. Silva, W. A., "Extension of a Nonlinear Systems Theory to General-Frequency Unsteady Transonic Aerodynamic Responses," AIAA 93-1590, Apr. 1993
14. Silva, W. A., "A Methodology for Using Nonlinear Aerodynamics in Aeroservoelastic Analysis and Design," AIAA 91-1110, Apr. 1991.
15. Lee, Y. W. and Schetzen, M., "Measurement of the Wiener Kernels of a Non-linear System by Cross-correlation," *International Journal of Control,* Vol. 2, No. 3, Sep. 1965, pp. 237-254.
16. Boyd, S., Tang, Y. S., and Chua, L. O., "Measuring Volterra Kernels," *IEEE Transactions on Circuits and Systems*," Vol. 30, No. 8, Aug. 1983, pp. 571-577.
17. Chadan, K., Colton, D., Päivärinta, L., and Rundell, W., *An Introduction to Inverse Scattering and Inverse Spectral Problems*, SIAM, Philadelphia, 1997.
18. Faller, W. E. and Schreck, S. J., "Unsteady Fluid Mechanics Applications of Neural Networks," AIAA 95-0529, Jan. 1995.
19. Faller, W. E., Schreck, S. J., Helin, H. E., and Luttges, M. V., "Real-Time Prediction of Three-Dimensional Dynamic Reattachment Using Neural Networks," AIAA 94-2337, 1994.
20. Faller, W. E., Schreck, S. J., and Luttges, M. W., "Real-Time Prediction and Control of Three-Dimensional Unsteady Separated Flow Fields Using Neural Networks," AIAA 94-0532, 1994.
21. Reisenthel, P. H., "Application of Nonlinear Indicial Modeling to the Prediction of a Dynamically Stalling Wing," AIAA 96-2493, Jun. 1996.
22. Cowan, T. J., Arena, A. S., and Gupta, K. K., "Accelerating CFD-Based Aeroelastic Predictions Using System Identification," AIAA 98-4152, 1998.