DEVELOPMENT OF A NONLINEAR INDICIAL MODEL USING RESPONSE FUNCTIONS GENERATED BY A NEURAL NETWORK

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ABSTRACT

In order to predict the dynamics of maneuvering aircraft or missiles at high rotational rates and high angles of attack, it is essential to accurately and efficiently model the nonlinearities associated with post-stall aerodynamics, including bifurcations and hysteresis. Nonlinear indicial theory offers a viable alternative which can fulfill the need for the efficient and accurate modeling of nonlinear "plant" characteristics. The present paper describes recent applications of linear and nonlinear indicial response models for aerodynamic prediction of maneuvering flight vehicles.

NOMENCLATURE

Symbols and abbreviations

b	Wing span
c	Wing chord
C_d	Sectional drag coefficient
C_D	Drag coefficient
C_L	Lift coefficient
C_l	Sectional lift coefficient
C_m	Sectional pitching moment coefficient
C_n	Sectional normal force coefficient
C_p	Pressure coefficient
C_t	Sectional tangential force coefficient
C_z	Vertical force coefficient
CSR	Critical-state Response
f	Aerodynamic load
f_α	Indicial response of f with respect to α
f_ϕ	Indicial response of f with respect to ϕ
h	Height with respect to mean sea level
H	Heaviside step function
IR	Indicial Response
k	Reduced frequency ($k \equiv \omega b/2U_\infty$)
q	Pitch rate
QS	Quasi-static
t	Time
t^*	Time at which the indicial step is applied
U_∞	Freestream velocity
α	Angle of attack
α^+	Nondimensional pitch rate ($\alpha^+ \equiv \frac{\dot{\alpha} c}{U_\infty}$)
λ	Wavelength
τ	Auxiliary time variable
τ_c	Time at which critical state is crossed
Δf	Aerodynamic load build-up
Δf^{CS}	Critical state response
ξ	Parameter denoting dependence on prior motion history
ω	Angular frequency

Subscripts

CS	Critical State
dyn	Dynamic
p	Predicted
qs	Quasi-static
s	Stall
∞	Time-asymptotic value (except for U_∞)

Superscripts

CS	Critical State
s	Stall
•	Derivative with respect to time
••	Second derivative with respect to time
~	Indicial function

1. PROBLEM DEFINITION

In recent years, it has been possible to integrate the flight-dynamics equations fairly efficiently using linearized aerodynamics which are occasionally

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supplemented with ad hoc methods (i.e., semiempirical simulations or wind tunnel data) to include nonlinear unsteady aerodynamic effects. However, with the expanded flight envelopes being considered for future maneuvering aircraft, it has become increasingly important to be able to model and predict nonlinear, unsteady aerodynamics. This includes the prediction of the aerodynamic response in the presence of flow separation, shock movement, and vortex bursting at high angles of attack and/or high angular rates.

Future fighter aircraft will be required to perform controlled maneuvers well beyond traditional aircraft limits, for example, pitch up and flight at high angles of attack, rapid point-to-shoot, and other close-in combat maneuvers. These advanced maneuvers demand the use of aerodynamic methods capable of predicting characteristics of the nonlinear post-stall regime for multiaxis motions at extremely high rates. At present, the only methods of this scope are Navier-Stokes methods. However, their use in flight simulations remains impractical at this time. Nonlinear indicial theory has the potential to circumvent some of the present difficulties, while providing a fidelity to the flow physics of which other methods appear incapable.

The present paper summarizes recent applications of unsteady aerodynamic modeling based on linear and nonlinear indicial theory. The aerodynamic indicial prediction depends on the availability of the indicial and critical state responses of the flow. The results of identification methods aimed at their extraction are described at the end of this paper.

2. OBJECTIVE

The overall objective of this work is the development of an aerodynamic modeling capability based on nonlinear indicial response theory. The results of earlier studies[1,2] have demonstrated the feasibility of building such a model using key simplifications based on the original formulation proposed by Tobak et al. (Ref. 3) and Tobak and Chapman (Ref. 4). Our goal is to validate this model by applying it to aerodynamic problems of increasing complexity.

First, some applications of linear indicial theory are briefly reviewed. This is followed by a summary description of the aforementioned feasibility study using nonlinear indicial theory. The paper ends with a description of ongoing work towards the systematic extraction of nonlinear indicial and critical state responses from unsteady aerodynamic databases.

3. INDICIAL THEORY

The indicial approach is based on the concept that a characteristic flow variable $f(t)$, which describes the state of the flow, can be linearized with respect to its boundary condition (or forcing function), $\epsilon(t)$, if the variation of $f(t)$ is a smooth function of $\epsilon(t)$. This allows the representation of $f(t)$ in a Taylor series about some value of $\epsilon = \epsilon_0$; thus

$$f(t) = f(0) + \Delta\varepsilon \left.\frac{\partial f}{\partial \varepsilon}\right|_{\varepsilon=\varepsilon_0} + \cdots$$

If the response $\partial f / \partial \varepsilon$ depends only on the elapsed time from the perturbation $\Delta\epsilon$ (a linear time invariant response) then it may be shown[5] that the formal solution for $f(t)$ is

$$f(t) = \tilde{f}(t)\,\varepsilon(0) + \int_0^t \frac{d\varepsilon}{d\tau}\,\tilde{f}(t-\tau)\,d\tau \quad (1)$$

where $\tilde{f}(t) = \left.\dfrac{\partial f}{\partial \varepsilon}\right|_{\varepsilon=\varepsilon_0}$.

Hence, if the forcing function (i.e., the boundary condition ϵ) is known and if $\tilde{f}$ (the indicial response) is known from some computation or experimental determination, then Eq. (1) gives the value of $f(t)$ for any schedule of boundary conditions $\epsilon(t)$ without the need to compute f from first principles.

The basic idea behind the use of *nonlinear* indicial functions,[3,4] is that the linear formalism, Eq. (1), can be retained in the form of a generalized superposition integral, provided that the nonlinear indicial response $\tilde{f}$ is now taken to be a functional $\tilde{f}(\epsilon(\xi);t,\tau)$, where $\epsilon(\xi)$ denotes the dependence on the entire motion history. Furthermore, the nonlinear indicial theoretical formulation allows for the presence of aerodynamic bifurcations by splitting the integral, i.e., for example:

$$\begin{aligned} f(t) = {} & f(\varepsilon(\xi);t,0) + \int_0^{\tau_c} \frac{d\varepsilon}{d\tau}\,\tilde{f}(\varepsilon(\xi);t,\tau)\,d\tau \\ & + \Delta f^{cs}(t;\varepsilon(\tau_c)) + \int_{\tau_c}^{t} \frac{d\varepsilon}{d\tau}\,\tilde{f}(\varepsilon(\xi);t,\tau)\,d\tau \end{aligned} \quad (2)$$

where the nonlinear indicial function $\tilde{f}(\epsilon(\xi);t,\tau)$ is defined as the following Fréchet derivative:

$$\tilde{f}(\boldsymbol{\varepsilon}(\boldsymbol{\xi});t,\tau) = \lim_{\Delta\boldsymbol{\varepsilon}\to 0} \frac{\Delta f(t)}{\Delta\boldsymbol{\varepsilon}} = \lim_{\Delta\boldsymbol{\varepsilon}\to 0}\left[\frac{f(\boldsymbol{\varepsilon}(\boldsymbol{\xi})+H(\boldsymbol{\xi}-\tau)\Delta\boldsymbol{\varepsilon})-f(\boldsymbol{\varepsilon}(\boldsymbol{\xi}))}{\Delta\boldsymbol{\varepsilon}}\right] \quad (3)$$

and $\Delta f^{CS}(t;\epsilon(\tau_c))$ is the so-called jump response associated with crossing the bifurcation at time τ_c.

A critical state is defined[6] as a transition from one equilibrium flow state to another and is often associated with a discontinuity in the static aerodynamic loads and/or their derivatives.[7] The associated transient response is referred to either as the critical state response (CSR) or the jump response, $\Delta f^{CS}(t;\epsilon(\tau_c))$.

4. LINEAR INDICIAL MODELING

Over the past several years, there have been a number of applications of linear indicial theory to either linear or linearized flow problems. These include supersonic flow, moving shocks, viscous flow (moving separation[8]), and aeroservoelasticity. The goal of this section is not to review these applications but, rather, to provide two illustrations. The first example[9] (application to supersonic flow) stresses the accuracy of the method. The second application (the simulation of a sea-skimming missile) illustrates the use of indicial theory in the context of a comprehensive multidisciplinary flight simulation.[10]

Figure 1 compares the numerically calculated lift response $\Delta C_L(t)$ to the ΔC_L predicted using indicial theory. The lift response corresponds to the hypothetical maneuver $\alpha(t)$ of a two-dimensional airfoil in inviscid flow at Mach 2.0 (Lesieutre et al., Ref. 9). Figure 1 indicates that the indicial function approach works extremely well for this flight condition; the indicial theoretical prediction (dotted line) is virtually indistinguishable from the numerically simulated response using ARC2D (dashed line). The error depicted in Fig. 1 is the difference between the actual (i.e., numerically computed) response and its analytic prediction based on indicial theory. The reason for the excellent agreement in this case is due primarily to the aerodynamics being linear for the low angle of attack conditions of the maneuver.

The development of a comprehensive multidisciplinary flight simulation tool based on indicial theory is described in Ref. 10. Conventional flight simulation programs generally model the aerodynamics of a vehicle by prespecifying the aerodynamic characteristics of the vehicle within a separate module. These characteristics are usually stored as either aerodynamic derivatives (for example, $C_{N\alpha}$) or as table look-up parameters as functions of flight conditions. This method of modeling the aerodynamics is fundamentally quasistatic and is usually augmented with aerodynamic damping derivatives such as C_{mq} to represent part of the unsteady effects. For flight low over the sea, as indicated in Fig. 2, the flow environment is typically unsteady, due to the presence of gusts from the waves and from atmospheric turbulence. Significant unsteady aerodynamic phase lags can occur under these adverse, often off-design, conditions which are not predicted by conventional methods. In addition, the accelerations experienced by the missile due to the unsteady flow environment may excite structural bending modes of slender missiles. Therefore, to model sea skimming flight over the sea, a method is required which models both the flexible missile and the unsteady aerodynamics.

Lesieutre et al. (Ref. 10) developed a new simulation program which models a flexible missile flying low over the sea and includes the effects of unsteady aerodynamics. The simulation contains detailed models of the missile guidance and control systems, including the sensors and fin actuators. Unsteady aerodynamic responses of the flight vehicle are computed off-line using CFD and integrated into the overall simulation using indicial theory. Typical simulation results at sea state 5 and Mach 0.8 are shown in Fig. 3. The missile is assumed to be flying in a direction perpendicular to the wave fronts and flies free for the first ten seconds after which it is commanded to fly low over the sea. Figure 3 gives the time histories for (from top to bottom) missile altitude, vertical velocity, pitch rate, tail fin deflection angle, and first bending mode amplitude.

5. NONLINEAR INDICIAL MODELING

An essential simplification of the nonlinear indicial formulation was achieved[1,2] by parameterizing the indicial and critical state responses using instantaneous motion parameters, for example α and $\dot{\alpha}$. The basic idea described in Refs. 1 and 2 is that, once the problem is appropriately parameterized, the knowledge of only a finite number of response functions may suffice to predict accurately the outcome of new maneuvers. While this may not be true of all aerodynamic systems, the simplifying assumption has not been contradicted by the two nonlinear aerodynamic systems examined thus far. The first model (referred to as the Goman-Khrabrov model, Ref. 11) is an analytical model which approximates the flight test aerodynamic responses of a fighter aircraft undergoing "Cobra"-type maneuvers. The application of nonlinear indicial theory to this first example (the Goman-Khrabrov model) is the topic of Ref. 1. This first example is useful in understanding the foundations of the nonlinear indicial prediction model. However, it is a simple nonlinear model, in the sense that there are no crossings of critical states. The second model is an artificial neural network which was trained on wind tunnel data of a pitching rectangular wing undergoing dynamic stall (Ref. 12). This second application is an example of a highly nonlinear system and includes at least one critical state (aerodynamic bifurcation), requiring special handling. The application of nonlinear indicial theory to the neural network example is described in more detail in Ref. 2.

The Goman-Khrabrov and neural network systems both exhibit complex nonlinear behaviors which approximate the unsteady flow physics. In each case, they are used (1) to generate the indicial and (if required) critical-state-response data, and (2) to compare with the indicial theoretical prediction for novel maneuvers. The application of the indicial theoretical prediction method to both systems is described in previous AIAA papers.[1,2] A brief summary of the results is given below.

5.1. Description of the Goman-Khrabrov System

The Goman-Khrabrov system is a mathematical model which was shown (Ref. 11) to accurately describe unsteady aerodynamic effects observed in several experimental investigations, including unsteady flow about an airfoil with trailing-edge separation (Ref. 13), a delta wing with vortex breakdown (Ref. 14, and a maneuvering fighter aircraft (Ref. 11). The model extends the usual flight dynamics equations by introducing a first order delay differential equation for an additional internal state variable x which accounts for unsteady effects associated with separated and vortex flow. The variable x may, for instance, represent the location of flow separation or that of vortex breakdown. The form of the differential equation governing x is:

$$\tau_1 \frac{dx}{dt} + x = x_0(\alpha - \tau_2 \frac{d\alpha}{dt}) \qquad (4)$$

where α is the angle of attack, x_0 describes the steady dependency of x on α, and τ_1 and τ_2 are time constants which must be determined using parameter identification techniques.

The system considered here is that of a full-scale fighter aircraft undergoing the well-known "Cobra" maneuver (Ref. 15), in which the vertical force coefficient C_z is related explicitly to the instantaneous values of x and α. This system was shown[16] to have no critical state. Note that the absence of aerodynamic bifurcations in the Goman-Khrabrov system pertains to the mathematical representation of the aerodynamics and does not necessarily imply their absence in the real flowfield. The main point is that nonlinear indicial theory was tested in this example, in isolation from any issues related to the inclusion of critical state (jump) responses.

5.2. Description of the Neural Network System

The second nonlinear flow model that was considered is an artificial neural network. This artificial neural network (described in the work of Faller et al., Refs. 12,17,18) was trained on wind tunnel data for a rectangular wing (NACA0015 profile, chord Reynolds number Re = 70,000) pitching about the 1/4 chord location. The particular neural network program that was used in this study is a recursive neural network, corresponding only to pitch up motions of the wing, from $\alpha = 0°$ to $\alpha = 60°$, and using a constant nondimensional pitch rate $0.01 < \alpha^+ < 0.2$. The neural network is treated here as a "black box" substitute for the experiment. The pitch-up neural network predicts the sectional force coefficients C_l, C_d, C_n, C_t, and C_m at three span locations (0%, 37%, and 80% of the total wing span), in addition to the pressure coefficients at 45 locations on the upper surface. The location of the pressure taps is shown in Fig. 4. The architecture of the neural network is schematically illustrated in Fig. 5. The neural network has an input layer with 47 inputs (45 fed back C_p's, plus the instantaneous α and $d\alpha/dt$), two hidden layers, each with 32 neurodes, and 60 outputs (45 predicted C_p's and 15 force coefficients).

This neural network system was shown (Ref. 12) to replicate not only the constant pitch rate data it was trained on, but also to be able to predict the flow responses to "novel maneuvers" (both constant and variable pitch rates) with surprisingly high accuracy, provided that the physics of the flow are similar. For our purpose, the trained neural network can be considered as a "black box" prediction method for the time-dependent loads and pressure distributions. This black box prediction is an accurate representation of the flow within a reasonably wide parameter space, and it is assumed that the flow responses predicted by the model are "as good" as if they were directly measured in wind tunnel tests.

5.3. Indicial Model Validation

There are two levels of testing of the indicial theoretical prediction. The first level (referred to as "validation") involves reproducing the very maneuver from which the indicial and critical-state responses were determined, prior to the indicial step. This so-called validation exercise examines the feasibility of smoothly blending a finite number of discrete indicial functions to predict the unsteady aerodynamic response *in the case where the indicial responses are known exactly*. By "known exactly" we mean that there is no error or uncertainty in the indicial function due to effects of prior motion history. By contrast, the indicial model prediction (Section 5.4) tests the accuracy of the prediction for *novel* maneuvers. This prediction is based on a finite number of prerecorded indicial and critical-state responses which do not share the same prior motion history as the maneuvers to be predicted.

An example of indicial model validation is shown in Figs. 6 and 7, corresponding to the Goman-Khrabrov and neural network systems, respectively. In Fig. 6, the data are the ΔC_z prediction of the Goman-Khrabrov model for a large amplitude Cobra maneuver over approximately six seconds (see Ref. 1). The indicial theoretical prediction (indicated by the dashed line) was obtained using 24 indicial responses. In Fig. 7, the data are the ΔC_l prediction of the neural network for a constant pitch rate maneuver, $\alpha^+ = 0.04$. In both figures, the quasistatic data are indicated for reference. Note that the near-perfect agreement between data and prediction in the latter case is due to two factors. The first is the high sampling of the indicial function space (one indicial function for every 0.275° in α, resulting in an order of magnitude more functions than in the Goman-Khrabrov case). The second is the prior identification of a critical state at the static stall angle ($\alpha_S \approx 16.4°$) and the accurate determination of the associated jump response (see Ref. 2). In practice, of course, it would be impractical to use so many indicial functions. The point of Figs. 6 and 7 is simply to validate the basic indicial theoretical frame work, Eq. (2).

5.4. Indicial Model Prediction

The true test of the indicial method lies in the ability to predict unsteady aerodynamic responses for completely new maneuvers. To achieve this, the indicial and critical state responses required during a given maneuver are approximated based on a finite set of prestored responses. In the present model, the prestored responses are required only to have *recent* time history characteristics which are similar to those of the maneuver to be predicted. For motions $\alpha(t)$ which are sufficiently smooth, the indicial function at time τ can be appropriately characterized using the instantaneous values of $\alpha(\tau), \dot{\alpha}(\tau), \ddot{\alpha}(\tau)$, etc. Thus, a central issue is to determine what constitutes an efficient parameterization of the indicial function space. The studies of Refs. 1 and 2 have shown that, for both the Goman-Khrabrov system and the neural network system, the indicial function space can be appropriately parameterized using only α and $\dot{\alpha}$.

Figures 8 and 9 indicate in parameter space the location of the indicial and critical state responses used for prediction, relative to the trajectories associated with various novel maneuvers. In Fig. 8 (Goman-Khrabrov model), 15 indicial responses have been precomputed, each corresponding to a constant pitch rate prior motion history (dotted lines). Figure 9 depicts the parameter space for the neural network system. In this case, the indicial prediction is based on 38 prerecorded indicial functions and three prerecorded critical state responses. Each response corresponds to a nominally constant pitch rate prior motion history ($\alpha^+ = 0.02$, 0.04, or 0.06).

The respective indicial theoretical predictions are shown in Figs. 10 and 11. In each case, both the indicial responses and the critical-state responses are functionally interpolated in $(\alpha, \dot{\alpha})$ parameter space, based on the precomputed *nodal* responses. Note that the prediction of the load build-up (ΔC_z for Goman-Khrabrov, Fig. 10, and ΔC_l for the neural network, Fig. 11) for a novel maneuver is a completely "hands-off" process: it does not require any prior knowledge of the critical-state response associated with that particular maneuver, nor does it require any knowledge of any of the indicial functions associated with it.

5.5. Discussion

Successful application of the nonlinear indicial method to both Goman-Khrabrov and neural network systems suggests that indicial theory, coupled with appropriate multivariate functional interpolation methods, could be used as a high angle-of-attack prediction method. In the study of Ref. 2 the indicial method was shown to be significantly more accurate than aerodynamic derivatives-based methods, which are not appropriate for true unsteady maneuvers, particularly when critical states are encountered. The method is also considerably faster than CFD since it involves only functional interpolation and the calculation of a generalized convolution integral. Thus, it has potential for future flight simulation applications.

6. DETERMINATION OF NONLINEAR INDICIAL AND CRITICAL STATE RESPONSES

One of the critical aspects of a nonlinear indicial prediction model is the availability of indicial and critical state responses. In the examples described in Section 5, the task of obtaining these responses was greatly facilitated by the nature of the models being considered, whether analytical (the Goman-Khrabrov system) or numerical (the neural network system). In practice, when dealing with experimental data (whether from wind tunnels or flight tests) the indicial and critical state responses are not readily available. Furthermore, the very nature of current dynamic wind tunnel tests is often ill-suited to the determination of these responses. The present Section describes some recent progress in the area of nonlinear indicial and critical state response extraction from aerodynamic databases.

Traditional methods of determining indicial responses include the direct method (described below), the Laplace domain method, and optimization/parameter identification methods. The direct method is based on strict adherence to the definition of the indicial response as a Fréchet derivative, Eq. (3). It involves performing a small step at some point in the maneuver. This method was used for the nonlinear problems described in Section 5. The Laplace method (Refs. 19,20) approximates the indicial response by taking an inverse Laplace transform after performing smooth approximations to a finite size step. This allows the alleviation of some of the difficulties associated with otherwise infinite accelerations.[9]

The direct method and the Laplace domain method are similar, in the sense that they require that step or step-like maneuvers be specifically carried out for the purpose of determining the indicial response. By contrast, optimization/parameter identification methods rely on the frequency scaling of the responses to infer the parameters of the indicial response (see, e.g., Ref. 21). This does require, however, that a certain form for the indicial response be assumed a priori, for instance, in potential flow, Theordorsen functions and the like.[22]

An alternative to these methods has recently been developed at Nielsen Engineering & Research. This alternative scheme uses projection methods, which result in the simultaneous solution of multiple nonlinear indicial and critical state responses. A key advantage of the new method is that it requires no prior assumptions about the functional form of the indicial or critical state responses and, most importantly, does not require that the maneuvers approximate steps. In the case where the maneuvers start from rest (start-up maneuvers), the determination of n indicial responses requires that n independent maneuvers be performed, and results in a linear system of simultaneous equations. If, additionally, p critical state responses must be extracted, then a total of on the order of $n+p$ maneuvers is required; however, this time, the system of equations is *a priori* nonlinear. In the particular case where the motions are periodic, a somewhat larger number of "training" maneuvers is required, and the answer (i.e., the solution vectors for the indicial and critical state responses) becomes approximate.

To illustrate how the method works, consider the following synthetically constructed nonlinear system. The input is the angle of attack, $\alpha(t)$. The output simulates an aerodynamic load and is denoted $f(t)$. It is assumed that the system is described exactly by a nonlinear indicial model with no critical state. The indicial responses at both ends of the interval ($f_{1,\alpha}$ at $\alpha = -1$ and $f_{2,\alpha}$ at $\alpha = +1$) are chosen to be exponentials and are shown in Fig. 12. The system is initially at rest ($\alpha = 0$, $\Delta f = 0$) and at time $t = 0$ a sinusoidal motion begins. Figure 12 shows that, using two such maneuvers, one can recover the nodal indicial responses $f_{1,\alpha}$ and $f_{2,\alpha}$ almost exactly. The accuracy of the indicial functions is confirmed by the good agreement between the indicial theoretical predictions $\Delta f_{1,p}$ and $\Delta f_{2,p}$ (obtained using the extracted indicial responses) and the "data" Δf_1 and Δf_2 (constructed using the theoretical indicial responses).

The second example (also based on synthetic data) is described below and corresponds to the simultaneous extraction of seven responses: five nonlinear indicial responses and two critical state responses. This synthetically constructed nonlinear system was designed to mimic the rolling moment coefficient response of a 65° delta wing undergoing forced roll oscillations. (This synthetic example is an intermediate step towards the application of the extraction method to the analysis of Wright Laboratory's 65−degree delta wing database.[6,7,23-25] Specifically, the subcase being simulated corresponds to small amplitude oscillations in the range $-4° \leq \phi \leq 8°$, for a support sting angle of 30° and at a freestream Mach number of three tenths. Static data taken at fine roll increments (see Refs. 7,25) suggest the existence in this range of critical state transitions at $\phi = 5.20°$ and $\phi = 4.67°$ for increasing and decreasing ϕ, respectively (see Fig. 13).

The real data is idealized using a nonlinear indicial model, which is illustrated in Fig. 14. It is <u>this</u> idealized model which is used to generate the training data (hysteresis loops) for the extraction method. In this manner, one can compare the extracted indicial and critical state responses to the actual (theoretical) ones, which are known in this case. The nonlinear indicial response model shown in Fig. 14 is based on a parameterization by ϕ only, with nodal indicial responses defined at roll angles equal to −4°, −1.3°, 1.6°, 5°, and 8.6°, respectively. Two jump responses are defined at the crossing of critical states, at $\phi_{CS} = 5.2°$ (for increasing ϕ), and at $\phi_{CS} = 4.7°$ (for decreasing ϕ). The various time constants were chosen so as to qualitatively reproduce some of the actual hysteresis loops recorded in this roll angle range (see, e.g., Fig. 15).

The training data set is generated by the nonlinear model, and consists only of periodic roll motions $\phi(t) = \phi_0 + \phi_1 \sin(kt)$, a subset of which is shown in Fig. 16. In this figure, f designates the simulated aerodynamic load (in this instance, a scaled representation of the rolling moment). The training data set consists of two groups of maneuvers: one centered at $\phi_0 = 0°$ and one centered at $\phi_0 = 3°$. In the first group, the maneuver amplitudes are ±2°, ±3°, and ±4°. In the second group, the maneuver amplitudes are ±2°, ±3°, and ±5°. For each amplitude, hysteresis loops may be available at as many as eight reduced frequencies: k = 0.010, 0.021, 0.031, 0.042, 0.052, 0.073, 0.084, and 0.105.

The result of the extraction procedure is shown in Fig. 17, where the seven simultaneously extracted nonlinear indicial and critical state responses are compared to the theoretical ones. There is good agreement between the two. Note that the noisy aspect of the extracted responses is a consequence of the large numerical stiffness of the systems being solved.

An alternative measure of the accuracy of the extraction process is to use the extracted indicial and critical state responses to *predict* the hysteresis loops associated with novel maneuvers. For example, let us consider the prediction of large amplitude maneuvers ($\phi = 2° \pm 8°$). Furthermore, let us use frequencies that were not used in the training data, e.g.: k = 0.0027, 0.0067, 0.0267, and 0.1467. Note that, of the four reduced frequencies, only one (k = 0.0267) is within the range of frequencies used in the training procedure. For the other three, the method effectively functions in extrapolation mode. The result of the prediction is shown in Fig. 18. The method is seen to accurately predict the hysteresis loops at all frequencies. Only the highest reduced frequency (k = 0.1467) is affected by high-frequency noise or error. At least part of this error is due to the noisy nature of the extracted responses; the other component is systematic error which appears at high frequency when using a fixed integration time step. The results shown in Fig. 18 correspond to the case where only 22 (of the possible 48) maneuvers were used as part of the training. Similar agreement was obtained with 31 and 48 maneuvers. The accuracy was found to degrade when using fewer frequencies. For example, with only nine training maneuvers, an average error of 10% was incurred in the prediction.

7. SUMMARY

Nonlinear indicial response theory addresses the need for high-fidelity prediction of nonlinear unsteady aerodynamic characteristics. Several application examples are presented, including the simulation of a fighter aircraft performing Cobra-type maneuvers, and a neural network simulating wing stall during pitch up maneuvers. Because of the analytical and numerical nature of these models, a direct determination of the nonlinear indicial and critical state responses was possible. However, the validation of the nonlinear indicial model using existing experimental data requires the use of specialized extraction/identification techniques. In this case, it may be possible to infer the nodal and critical state responses using a training data set. The inference mechanism thus uses a methodology which bears some similarity with the training of a neural network, although any further connection remains to be explored. One essential difference between neural networks and the present indicial theoretical model is the potential of the latter to be physics-based, since the nonlinear indicial response can be derived under certain conditions[26] from the Navier-Stokes equations.

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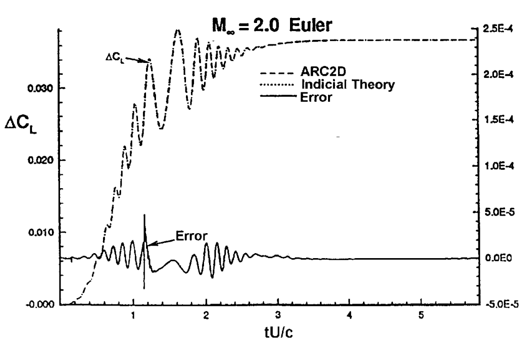

Fig. 1. Time Domain Comparison of Indicial Theoretical Prediction Versus Numerical Calculation (Ref. 9).

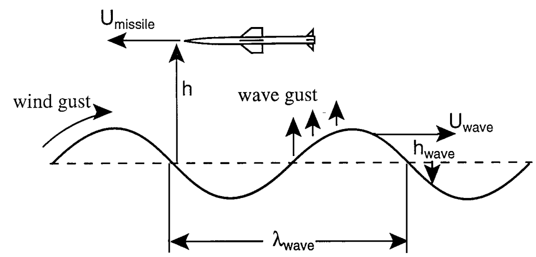

Fig. 2. Sea Skimming Missile Geometry and Flight Profile (Ref. 10).

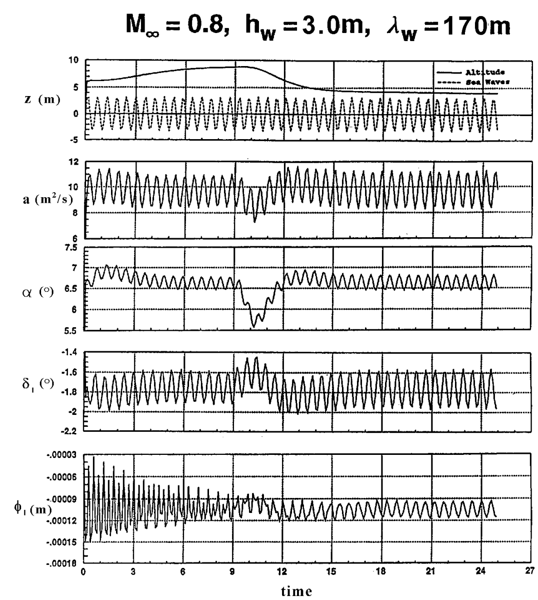

Fig. 3. Flexible Simulation Results for $M_\infty = 0.8$ and Sea State 5 (Ref. 10).

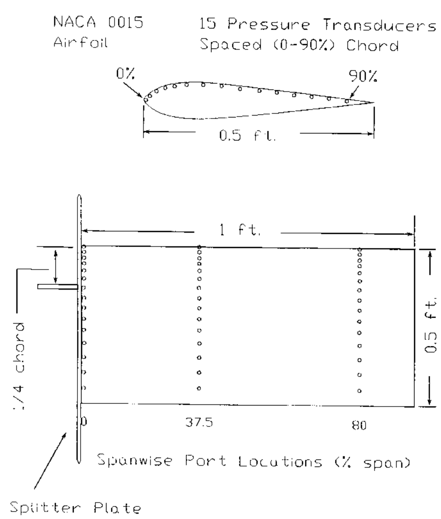

Fig. 4. Schematic Illustrating the Rectangular Wing Used to Train the Artificial Neural Network. (from Ref. 17).

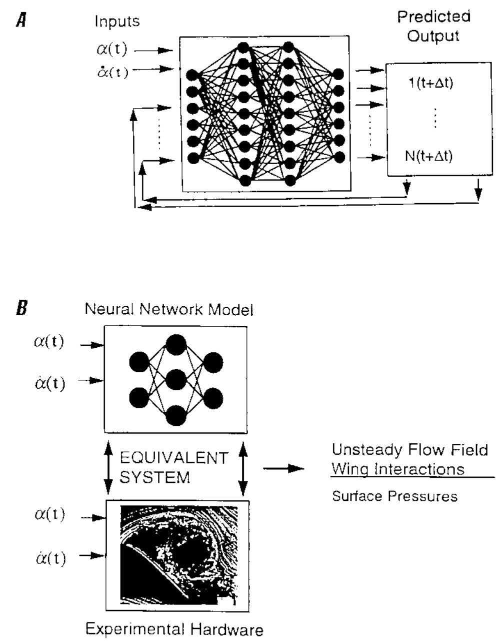

Fig. 5. (A) Schematic of the Neural Network Architecture. (B) The Operational Neural Network Following Training (Fixed Weights). (Adapted from Ref. 18, with permission).

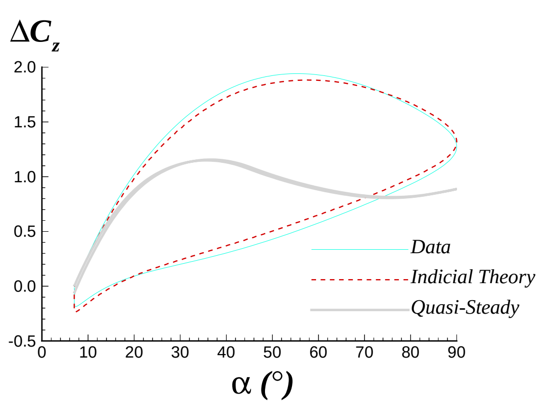

Fig. 6. Hysteresis Plot of Indicial Theoretical Prediction Versus Data (Large Amplitude Cobra Maneuver, Ref. 1).

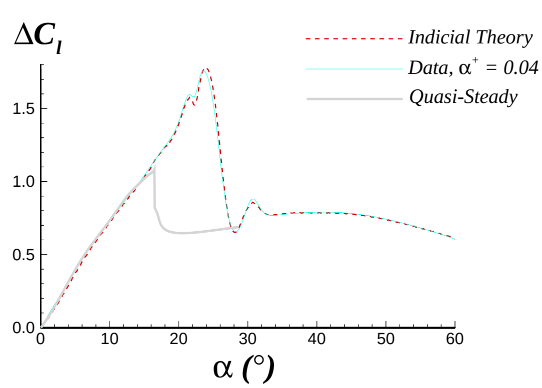

Fig. 7. Lift Indicial Theoretical Prediction Versus Data, $\alpha^+ = 0.04$ (Ref. 2).

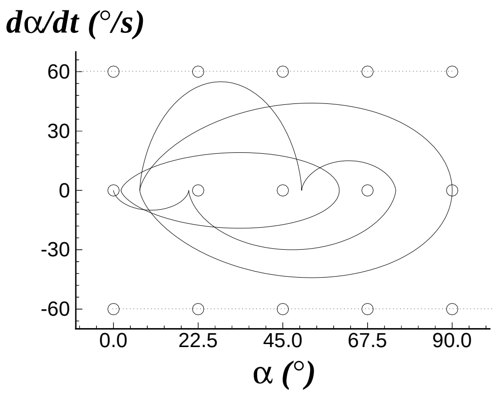

Fig. 8. Illustration of Maneuver Trajectories in Parameter Space (Ref. 1). Open symbols indicate the location of the nodal indicial functions.

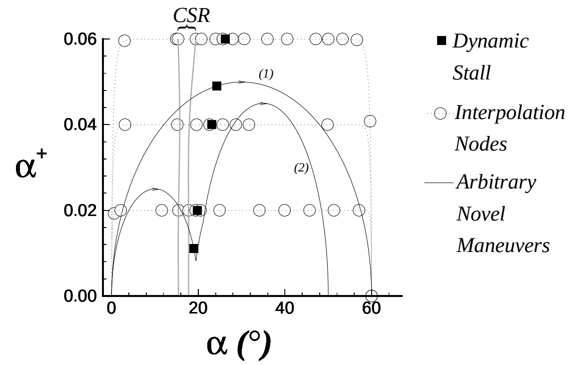

Fig. 9. Parameter Space Representation of Two Novel Maneuvers Illustrating Their Trajectory in Relation to the Location of the Nodal Indicial Responses (Ref. 2).

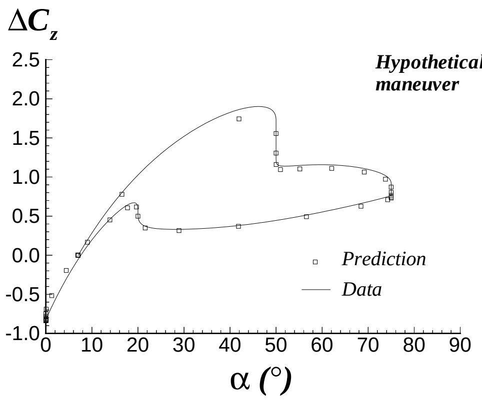

Fig. 10. Nonlinear Indicial Prediction Versus Data for Novel Maneuver (Ref. 1).

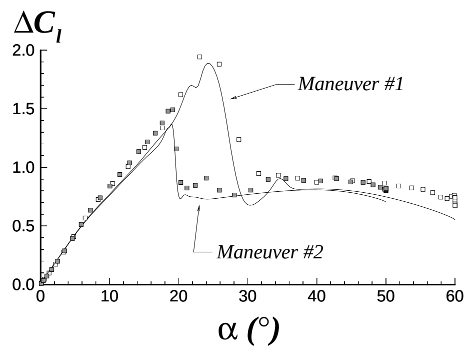

Fig. 11. Nonlinear Indicial Prediction (Symbols) Versus Data (Lines) for Two Novel Maneuvers (Ref. 2).

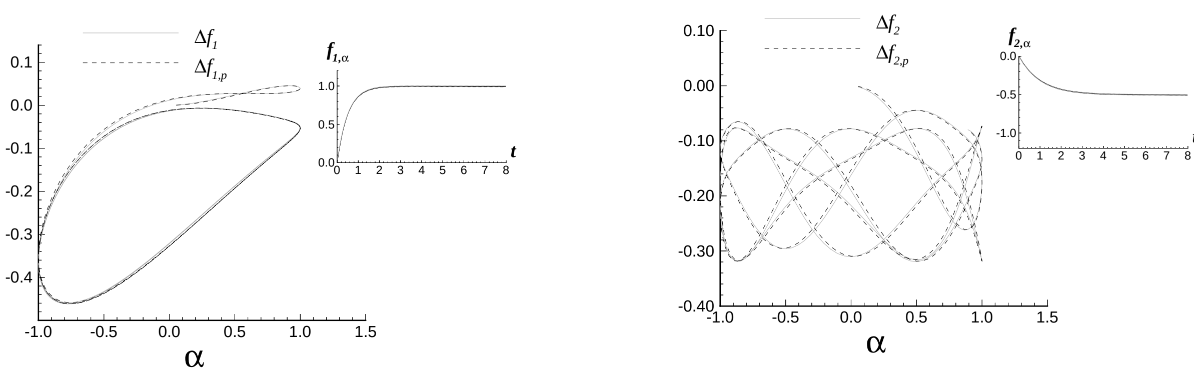

Fig. 12. Comparison of Predicted Aerodynamic Response $\Delta f_p(\alpha)$ Versus Actual Response Δf (Left: $\omega = 5\pi/6$, Right: $\omega = 2\pi$). Theoretical (dotted line) and extracted (solid line) responses are superposed in each inset.

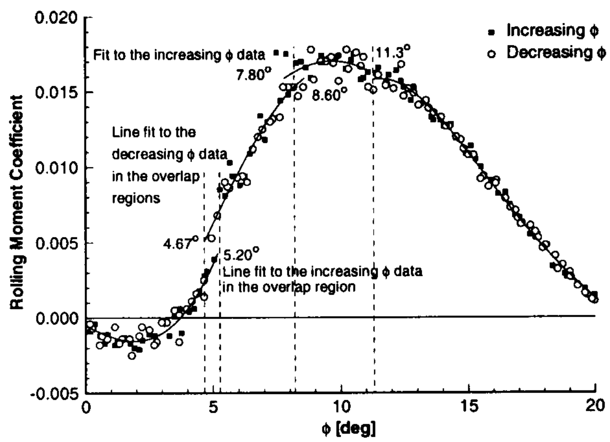

Fig. 13. Time-Averaged Static Rolling Moment Coefficient for both Increasing and Decreasing ϕ (from Ref. 25).

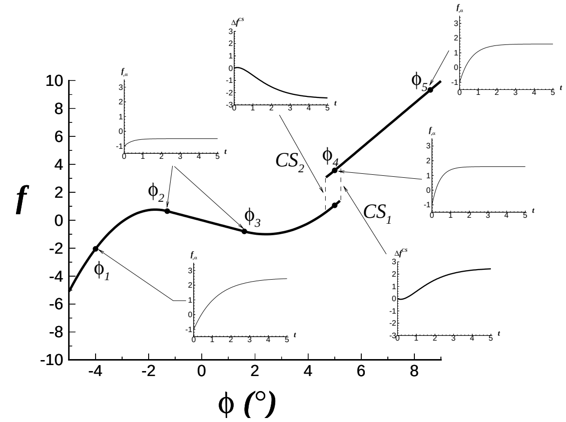

Fig. 14. Nonlinear Indicial Model Constructed Using Five Nodal Indicial Responses (Roll Angles ϕ_1, ϕ_2, ϕ_3, ϕ_4, and ϕ_5), and Two Critical State Responses (CS_1 and CS_2). (The thick solid line represents the quasistatic response).

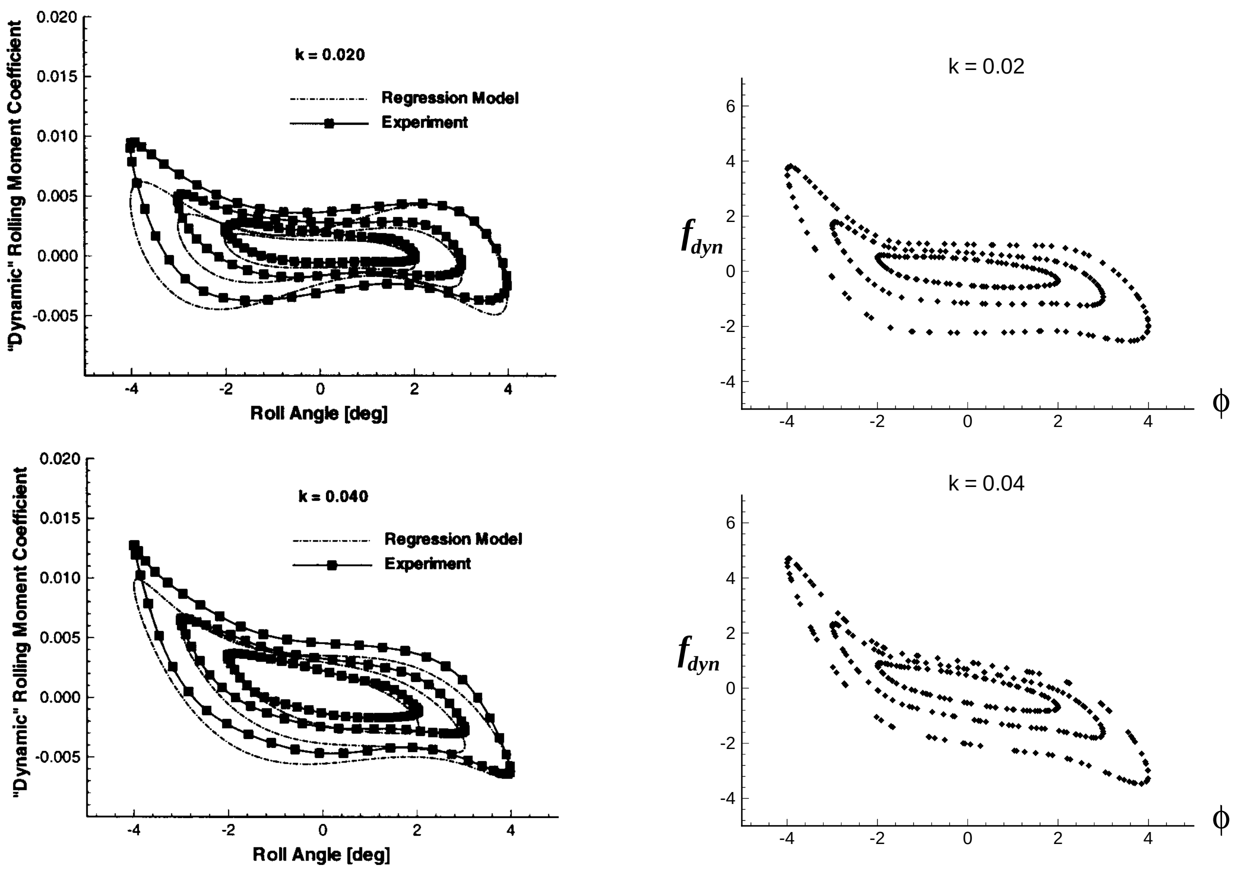

Fig. 15. "Synthetic" Hysteresis Loops (Right) Constructed Using the Model of Fig. 14, Versus Actual Hysteresis Loops (Left, from Ref. 25) for the Dynamic (Total Minus Static) Rolling Moment Coefficient for Harmonic Oscillations Centered at ϕ_0 = 0°. (Top: k = 0.02, bottom: k = 0.04).

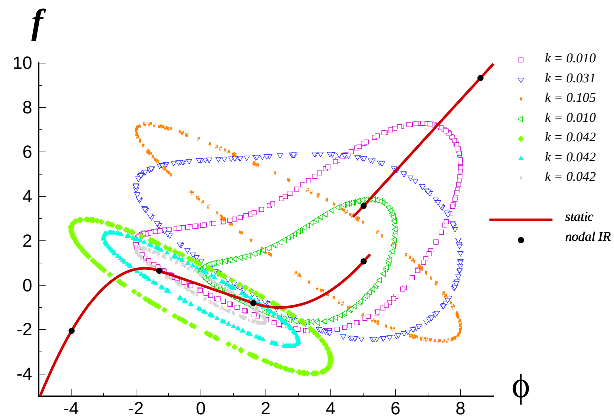

Fig. 16. Example of Training Data Subset Constructed Using the Model of Fig. 14.

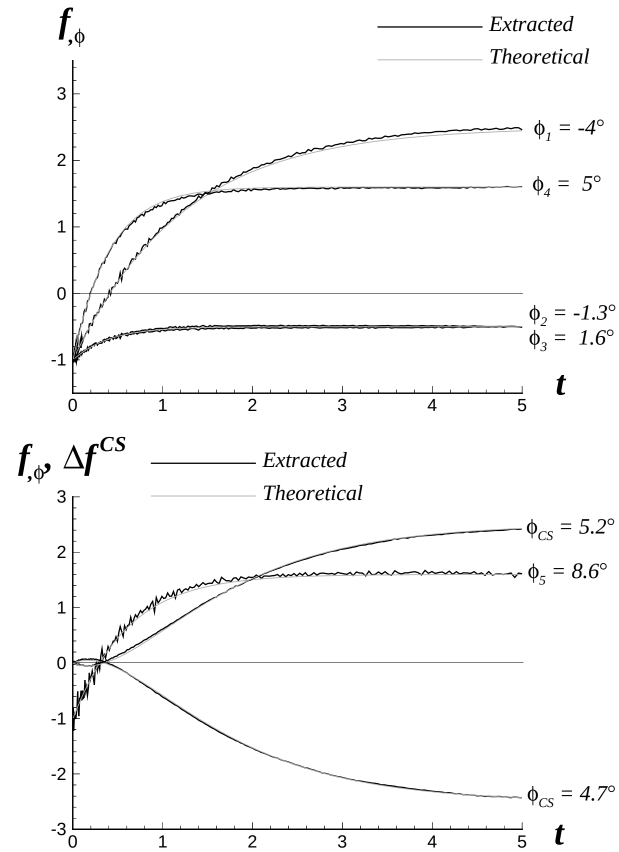

Fig. 17. Comparison of Extracted Versus Theoretical Indicial and Critical State Responses.

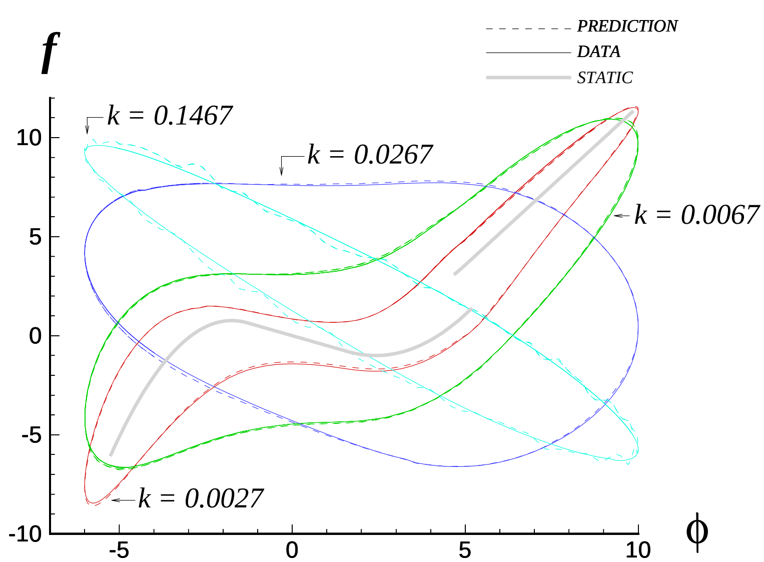

Fig. 18. Indicial Theoretical Predictions (Dashed Lines) Versus Data (Solid Lines) for Four Novel Maneuvers. (Number of maneuvers in the training set is 22).